APPLICATION OF GOAL PROGRAMMING IN FARM AGRICULTURAL PLANNING

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ABSTRACT

In this paper we present an overview of the different goal programming formulations, their assumptions, limitations and implications for agricultural decision making. The concept of standardized dual variables which provides a more meaningful interpretation of shadow prices in goal programming is introduced through a simple example of farm agricultural planning.
Keywords: Goal Programming, Decision Making, Farm Planning.

INTRODUCTION

Goal programming technique has its great potential, particularly in decision-making environments involving multiple objectives like farm agricultural planning and management.

In many situations farmers are often faced with several objectives simultaneously and no easy single choice. Examples of such objectives are: maximization of net revenue; minimization of capital borrowing and hired labour; and minimization of the risk associated with yield and field day variability. On a regional or national level, the agricultural decision maker may be faced not only with decisions about economic growth, but also about population nutritional requirements, strategic planning, environmental and other institutional issues. In this paper, we present aspect of goal programming based on duality theory through an example of farm agricultural planning.

GOAL PROGRAMMING MODEL

The importance of more than one objective in agricultural planning, studies is illustrated by the works of Ignizio et al., Barnett et al., Candler and Boehlje., Harper and Eastman., Hatch et al., and Willis and Perlack. Romero., and Schniederjans provide excellent reviews of the development in goal programming.

A generic-type goal planning model can be compactly written as:

Minimize: $\quad w^+ d^+ + w^- d^-$ $\qquad$ .... (1)

Subject to: $\quad Gx + d^+ - d^- = g$ $\qquad$ .... (2)

$\qquad Ax \qquad\qquad \leq b$ ....(3)

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$$x, \ d^+, \ d^- \geq 0 .... (4)$$

where x is a (n×1) vector of decision variables, G is a (p×n) matrix of goal contribution coefficients; g is a (p×1) vector of desired goal levels; d+ and d– are (p×1) vector representing, respectively, positive and negative deviations from goals; A is a (m×n) matrix of technological coefficients; b is a (m×1) vector of resource levels. In addition, w+ and w– are (1×p) vectors of weights which may or may not be preemptive.

DUALITY INTERPRETATION OF GOAL PROGRAMMING

When a goal programming is solved as a minimum sum of weighted deviations, the problem of choosing the weights has been the focus of a lot of work because of its complexity. The commonly used weighting procedure is to have the decision maker associate the highest weights with the most important goals. The use of duality theory may help provide a new insight into the design and interpretation of time weights. We focus on the interpretation of duality in goal programming and its usefulness in decision making.

Without loss of generality we restrict our attention to a primal problem with only goal constraints. In addition, the distinction between preemptive and non-preemptive formulations will be implicit in the distinction between a single dual or a sequence of prioritized duals.

Consider the following primal goal programming and dual goal programming formulations in compact form:

Primal :   Minimize: $Z = w^+ d^+ + w^- d^-$

Subject to: $Gx + d^+ - d^- = g$

$$x, \ d^+, \ d^- \geq 0$$

Dual :   Maximize: $y = vt \ g$

Subject to: $G^t v \leq 0$

$$-w^- \leq v \leq w^+$$

where v is a vector of dual variables, y is the value of the dual objective function, the problem dimensions are as in (1) – (4), and t is used to indicate transpose.

In goal programming, the primal may still be interpreted in physical terms where an optimal product mix contributes to the achievement of a certain number of goal targets. However, the corresponding dual is a "pricing problem of the goal targets" in terms of the decision maker's preferences. Hence, the dual should have preference–utility interpretation with the dual variables representing absolute marginal utilities of the different goals. We show below that a more useful interpretation of the dual variables is in relative terms, and we provide a numerical illustration.

From primal-dual relationships between primal goal programming and dual goal programming, we have the following:

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y < z (for any pair of feasible solutions) ....(5)

y* = z* (at optimality)..(6)

$$\frac{\partial z^*}{\partial g_i} = v_i^*$$

.... (7)

$$\frac{\partial y^*}{\partial d_i^*} = w_i$$

.... (8)

The first two relations can be interpreted together as follows: the total disutility of deviation from the goal targets (i.e., z) is always at least as large as the total utility of these goal targets (i.e., y), and equal to it at optimility.

Relation (7) states that the ith optimal dual variable represents the marginal effect of the ith goal target of total disutility (or utility). Relation (8) is a restatement of the weights attached to deviations (either positive or negative) as marginal effects of these deviations. Finally, from (7) and (8) we also obtain the following relations:

vi* (∂gi) = wi (∂di*) = ∂z* = ∂y* .... (9)

vi* / wi = ∂di* / ∂gi ....(10)

from (9) we see that the same marginal disutility / utility effect can be derived from either changing the goal targets or changing the corresponding deviations. But the most interesting result is given by (10) which defines a "marginal rate of substitution" between a goal target and the distance away from it as equal to the marginal utility of the goal divided by its weight. Hence, we can use these standardized dual variables (vi*/wi) as a measure of goal achievement across all goals. This provides a way of properly interpreting and using the dual variables in goal programming with or without preemption.

NUMERICAL EXAMPLE

The following simplified example is used throughout to illustrate the analysis. We have chosen two variables and two constraints so that we may be able to have graphical interpretations for both the primal and the dual. Consider a farmer who can grow either Wheat or Gram. He has 20 acres of land and would like to achieve at least Rs. 6,00,000 of revenue while minimizing total production cost. Table 1 contains the relevant information.

Table 1 : Cost / Revenue Data

	Wheat	Gram
Yield cwt/Acre	20	8
Price Rs/cwt	1100	3200
Cost Rs / Acre	8,000	7000

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This problem is formulated as the following linear programming problem:

(LP) Minimize: $z = 8,000x1 + 7,000x2$

subject to: $x1 + x2 < 20$

$22,000x1 + 25,600x2 > 6,00,000$

$x1, x2 > 0$

where x1 and x2 represents acres of wheat and gram respectively. Clearly, the problem has no feasible solution, as shown in figure 1. To get around the infeasibility and still find an acceptable "compromise" solution, the following simple goal programming formulation is used:

(GP) Minimize: $z = 1 \quad \square \quad \left(\dfrac{d_1^-}{20}\right)100 + \alpha_2\left(\dfrac{d_2^+}{6,00,000}\right)100$

Subject to: $x_1 + x_2 + d_1^+ - d_1^- = 20$

$22,000x1 + 25,600x2 + d_2^+ - d_2^- = 6,00,000$

$x1, \ x2, \ d^+, \ d^- > 0$

where d1 and d2 are weights attached to the deviation variables. In this form the objective function is expressed as a weighted sum of percent deviations from targets. For illustration purposes a particular set of weight is d1 = d2 =1 meaning equal importance, is given to both goals. We let

$\dfrac{100\alpha_1}{20} = w_1$; and $\dfrac{100\alpha_2}{6,00,000} = w_2$

This formulation is easily interpreted in light of Figure 1 as one which seeks to minimize the total infeasibility in the constraints of the original linear programming problem. The resulting goal programming solution is x2 = 20 (grow 20 acres of gram) and d2+ = Rs. 88,000 (revenue shortage) and corresponds to P1 in Figure 1. For the farmer this solution will guarantee as Rs 5,12,000 total return.
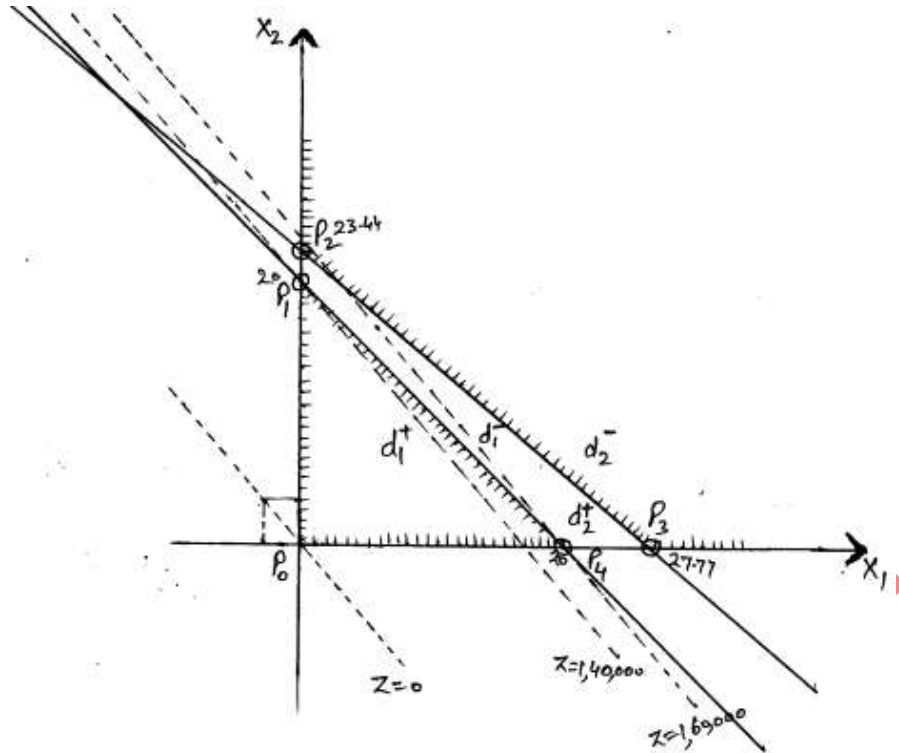
Figure 1: Solution of Farm Example

Now, suppose that the farmer looks at this solution and requires that an absolute first priority is not violate the revenue constraint because it may be a bankruptcy level. Once this is achieved, a minimization of the infeasibility of the total land constraint will be sought. This requirement is formulated as a lexicographic (or prioritized) goal programming with objective function $\{(d_2^+),(d_1)\}$ solved in two iterations:

Iteration One:

Minimize: $z = \{d_2^+\}$

Subject to : $22{,}000x1 + 25{,}600x2 + d2{+} - d2{-} = 6{,}00{,}000$

$x_1, \quad x_2, \quad d_1^+, \quad d_1^- \quad \geq 0$

with optimal solution:

$$d_2^+ = 0,\ 22{,}000x1 + 25{,}600x2 - d_2^{\;-} = 6{,}00{,}000$$

(Line segment P2P3 in Figure 1)

Iteration Two :

Minimize: $z = \{d_1^-\}$

Subject to: $x_1 + x_2 + d_1^+ - d_1^- = 20$

$22{,}000x1 + 25{,}600x2 + d_2^{\;-} = 6{,}00{,}000$

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$$x_1, \ x_2, \ d_1^+, \ d_2^+, \ d_2^- \ \geq \ 0$$

with optimal solution :

$$x2 = 23.44, \qquad d_1^- = 3.44 \qquad \text{(P2 in Figure 1)}$$

The case of an optimal but dominated solution, due to alternative optima, can also be illustrated. Consider the case where the farmer wants to use up all the land available (20 acres). The corresponding goal programming will have objective function $\{d_1 \, d_1^+\}$ ⊩. A regular simplex code gives the following solution: x1 = 20, d_2^+ = 1,60,000 (P4 in Figure 1). But an alternate optimal solution is easily found to be: x2=20, d_2^+ = 88,000 (P1 in Figure 1). In terms of the original problem the two solutions are compared in Table 2, from which clearly the first solution is dominated.

Table 2 : Comparing Alternate GP Solutions

	Optimal Solution $x_1 = 20$; $x_2 = 0$	Alternate Optimal Solution $x_1 = 0$; $x_2 = 20$
Cost	1,60,000	1,40,000
Revenue	4,40,000	5,12,000
Deficit	1,60,000	88,000

To illustrate the duality result, consider the following goal programming model corresponding to the original linear programming problem, which is inconsistent:

(GP)   Minimize : $z = w_1^- d_1^- + w_2^+ d_2^+$

Subject to : $x_1 + x_2 + d_1^+ - d_1^- = 20$

$$22{,}000x_1 + 25{,}600x_2 + \ d_2^+ - d_2^- = 6{,}00{,}000$$

$$x_1, \ x_2, \ d^+, \ d^- \geq \ 0$$

where w_1^- and w_2^+ are chosen in a way that will make the objective function consistent and should reflect the importance of each goal. If we use the opportunity costs of incurred shortages as a measure of relative importance, then $w_1^- = 1{,}27{,}000$ is interpreted as the cost of providing an additional acre of land and $w_2^+ = 1.00$ is the cost per unit of additional revenue foregone.

Now, we consider the dual problems of both linear programming and goal programming above:

(DLP) Maximize: $V = -20y_1 + 6{,}00{,}000y_2$

Subject to: $-y_1 + 22{,}000y_2 \leq 8{,}000$

$$-y_1 + 25{,}600y_2 \leq 7{,}000$$

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$$y_1, \, y_2 \geq 0$$

(DGP) Maximize: $V = -20y_1 + 6,00,000y_2$

Subject to: $-y_1 + 22,000y_2 \leq 0$

$$-y_1 + 25,600y_2 \leq 0$$

$$0 \leq y_1 \leq 1,27,000$$

$$o \leq y_2 \leq 1$$

Figure 2, depicts both problems with the hached area corresponding to dual linear programming and the cross-hatched area corresponding to dual goal programming.

Since the original linear programming problem was inconsistent, by duality theory, we know that its dual linear programming is unbounded. The goal program which was formulated to resolve the inconsistency is a linear program which has feasible solutions. Its dual goal program has the finite optimal solution: $y1 = 25,600$, $y2 = 1$ as can be seen from Figure 2. This solution corresponds to shadow prices of Rs. – 25,600 for land and Rs. 1.00 for total revenue, with the following interpretations, based on (5) – (10) above:
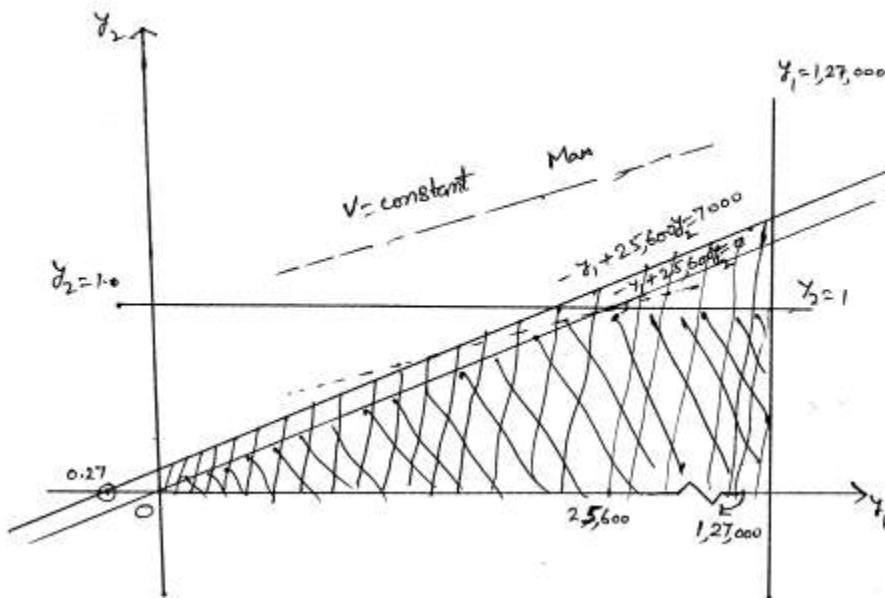

Figure 2 : Dual Solutions for (DLP) and (DGP)

(1)   In absolute terms, the magnitude of the shadow prices can be misleading in terms of the ranking of the constraints with respect to total goal achievement. That is, an additional acre of land will improve the total goal achievement by Rs. 25,600 even though it may cost at least Rs. 1,27,000 to acquire; on the other hand, an additional rupees of revenue forgone will only improve the total goal achievement by one rupee. The first part of Table 3 summerizes these effects.

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(2)   In relative terms, when comparing the achievement of both goals, the ratios of the shadow prices to their corresponding primal weights (standardized dual values) convey more meaningful information:

$$\frac{y_1}{w_1^-} = \frac{25{,}600}{1{,}27{,}000} = 0.20 \quad and \frac{y_2}{w_2^+} = \frac{1}{1} = 1.00$$

These ratios reveal that on a per unit basis total revenue contribute 100 percent to goal achievement compared to 20 percent for land. This is can be seen from the fact that the Rs 1,27,000 opportunity cost of land will yield Rs. 1,27,000 improvement instead of Rs. 25,600 for land (see table 3, second part). This suggests that care should be taken in practically interpreting shadow prices in goal programming.

Table 3: Shadow Price Effects

Constraint 1 (Acre)	Constraint 2 (Rs.)	Total Goal Achievement V (Rs.)	Change in V (Rs.)
20	6,00,000	88,000	– –
21	6,00,000	62,400	– 25,600
19	6,00,000	1,13,600	+ 25,600
20	6,00,001	88,001	+1
20	5,99,999	87,999	-1
20	7,27,000	2,15,000	+1,27,000
20	4,73,000	– 39,000	–1,27,000

CONCLUSION

The goal programming approach are two problems embodied in the following questions:
(a) Priorities: How to associate the results of a given solution to the satisfaction of the ranking; (b) Weights: How to generate them and what they mean. We think that the duality interpretation of the weights can help both the analysts and the decision makers in the design and solutions of meaningful multicriteria decision problems in farm agricultural planning and management.

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